

Introduction

[ETAS](#) is the acronym for the “Elementary Theory of Abstract Supercategories” as defined by the [axioms of metacategories and supercategories](#).

The following are simple examples of [supercategories](#) that are essentially interpretations of the eight [ETAC](#) axioms reported by W. F. Lawvere (1968), with one or several ETASaxioms added

as indicated in the examples listed. A family, or class, of a specific level (or 'order') $(n + 1)$ of

a supercategory $\S_{n+1}$ (with n being an integer) is defined by the specific [ETAS axioms](#) added to the eight [ETAC axioms](#); thus, for $n = 0$, there are no additional ETAS axioms and the

supercategory $\S_1$ is the limiting, lower [type](#), currently defined as a [category](#) with only one [composition law](#) and any standard interpretation of the eight ETAC axioms. Thus, the first

level of 'proper' supercategory $\S_2$ is defined as an interpretation of ETAS axioms S1 and S2;

for $n = 3$, the supercategory $\S_4$ is defined as an interpretation of the eight ETAC axioms plus the additional three ETAS axioms: S2, S3 and S4. Any (proper) recursive [formula](#) or '[function](#)'

can be utilized to generate supercategories at levels n higher than $\S_4$ by adding [composition](#) or consistency laws to the ETAS axioms S1 to S4, thus allowing a digital [computer algorithm](#) to

generate any finite level supercategory $\S_n$ syntax, to which one needs then to add semantic interpretations (which are complementary to the computer generated syntax).

Simple examples of ETAS interpretation in supercategories

1. [functor categories](#) subject only to the eight ETAC axioms;
2. [functor supercategories](#), $\mathcal{F}_S : \mathcal{A} \rightarrow \mathcal{B}$, with both $\mathcal{A}$ and $\mathcal{B}$ being '[large](#)' [categories](#) (i.e., $\mathcal{A}$ does not need to be small as in the case of functor categories);
3. A [topological groupoid](#) category is an example of a particular supercategory with all invertible [morphisms](#) endowed with both a [topological](#) and an algebraic structure, still subject to all ETAC axioms;
4. [supergroupoids](#) (also definable as [crossed complexes](#) of [groupoids](#)), and [supergroups](#) - also definable as [crossed modules](#) of groups- seem to be of great interest to mathematicians currently involved in 'categorified' [mathematical physics](#) or [physical mathematics](#).)
5. A [double groupoid](#) category is a 'simple' example of a higher dimensional supercategory which is useful in [higher dimensional homotopy](#) theory, especially in [non-Abelian algebraic topology](#); this [concept](#) is subject to all eight ETAC axioms, plus additional

axioms related to the definition of the double groupoid (generally [non-Abelian](#)) structures;

6. An example of 'standard' supercategories was recently introduced in mathematical (or more specifically 'categorified') physics, on the web's [n-Category café's web site](#) under "Supercategories". This is a rather 'simple' example of supercategories, albeit in a much more restricted sense as it still involves only the standard categorical homomorphisms, homo-functors, and so on; it begins with a somewhat standard definition of [super-categories](#), or 'super categories' from [category theory](#), but then it becomes more interesting as it is being tailored to [supersymmetry](#) and extensions of '[Lie](#)' [superalgebras](#), or [superalgebroids](#), which are sometimes called graded '[Lie](#)' [algebras](#) that are thought to be relevant to [quantum gravity](#) ([6] and references cited therein). The following is an almost exact quote from the above [n-category](#) café's website posted mainly by Dr. Urs Schreiber: A supercategory is a diagram of the form:

$$\diamond \diamond Id_C \diamond C \diamond \diamond s$$

in Cat-the category of categories and (homo-) functors between categories- such that:

$$\diamond \diamond Id \diamond \diamond Id_C \diamond C \diamond C \diamond \diamond s \diamond \diamond s = \diamond \diamond Id_C \diamond Id_C \diamond \diamond Id,$$

(where the 'diamond' symbol should be replaced by the symbol '[square](#)', as in the original Dr. Urs Schreiber's postings.)

This specific instance is that of a supercategory which has only one [object](#)- the above quoted [superdiagram](#) of diamonds, an arbitrary abstract category C (subject to all ETAC axioms), and the standard category [identity](#) (homo-) functor; it can be further specialized to the previously introduced concepts of supergroupoids (also definable as crossed complexes of groupoids), and supergroups (also definable as crossed modules of [groups](#)), which seem to be of great interest to mathematicians involved in 'Categorified' mathematical physics or physical mathematics.) This was then continued with the following interesting example. "What, in this sense, is a braided monoidal supercategory?". Dr. Urs Schreiber, suggested the following answer: "like an ordinary braided monoidal category is a 3-category which in lowest degrees looks like the trivial 2-group, a braided monoidal supercategory is a 3-category which in lowest degree looks

$$G(2) = (\diamond 2 \diamond Id \diamond 2) \quad \text{''}.$$

like the strict 2-group that comes from the crossed module

$$G(2)$$

Urs called this generalization of stabilization of n-categories, $G(2)$ -stabilization. Therefore, the claim would be that 'braided monoidal supercategories come

from $G(2)$ -stabilized 3-categories, with $G(2)$ the above strict 2-group';

7. An [organismic set](#) of order n can be regarded either as a category of [algebraic](#) theories

$$o \leq n$$

representing [organismic sets](#) of different orders or as a discrete topologyorganismic supercategory of algebraic theories (or supercategory only with discrete topology, e.g. , a class of objects);

8. Any [`standard' topos](#) with a (commutative) [Heyting logic algebra as a subobject classifier](#) is an example of a commutative (and distributive) supercategory with the additional axioms to ETAC being those that define the Heyting logic algebra;

$$LM_n$$

9. The generalized (Łukasiewicz- Moisil) toposes are supercatgeories of non-commutative, algebraic n -valued logic [diagrams](#) that are subject to the axioms

$$LM_n$$

of LM_n algebras of n -valued logics;

10. n -categories are supercategories restricted to interpretations of the ETAC axioms;
11. An organismic supercategory is defined as a supercategory subject to the ETAC axioms and also subject to the ETAS axiom of complete self-reproduction involving π -entities (viz. Löfgren, 1968; [1]); its objects are classes representing organisms in terms of morphism (super) diagrams or equivalently as heterofunctors of organismic classes with variable [topological structure](#);

Definition 0.1 Organismic Supercategories ([1]) An example of a class of supercategories interpreting such ETAS axioms as those stated above was previously defined for [organismic structures](#) with different levels of [complexity](#) ([1]); organismic supercategories were thus defined as superstructure interpretations of ETAS (including ETAC, as appropriate) in terms of

$$K = (C, \Pi, N)$$

triples , where C is an arbitrary category (interpretation of ETAC axioms, formulas, etc.), Π is a category of complete self-reproducing entities, π , ([4]) subject to the negation of the axiom of restriction (for elements of

$$\exists S : (S \neq \emptyset) \text{ and } \forall u : [u \in S] \Rightarrow \exists v : (v \in u) \text{ and } (v \in S)]$$

sets): , (which is known to be independent from the ordinary logico-mathematical and biological reasoning), and N is a category of non-atomic expressions, defined as follows.

Definition 0.2

$$\pi \pi \langle \pi \rangle$$

An atomically self-reproducing entity is a unit class [relation](#) u such that , which means

$$\pi \pi \langle \pi, \pi \rangle$$

“ π stands in the relation π to π ”, , etc.

An expression that does not contain any such atomically self-reproducing entity is called a non-atomic expression.

Bibliography

See references [13] to [26] in the Bibliography for Category Theory and Algebraic Topology

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